A Z score can also be derived using Newton's iterations and equation 4 as follows:

Define $P = \phi(z)$, and then from equation 4:

$$\phi(z) \cong 1 - \left(1 - \frac{1}{2z^2}\right)\frac{1}{z\sqrt{2\pi}}e^{-z^2/2}$$

let $\quad G(z) = \phi(z) - p$

Substituting the expression of $\phi(z)$ into G(z) results in equation 5:

EQN (5) $$G(z) = 1 - p - \left(1 - \frac{1}{2z^2}\right)\frac{1}{z\sqrt{2\pi}}e^{-z^2/2}$$

$$= \xi - \left(1 - \frac{1}{2z^2}\right)\frac{1}{z\sqrt{2\pi}}e^{-z^2/2}$$

Taking the first derivative of G(z) with respect to z results in:

$$\frac{dG(z)}{dz} = \frac{d\phi(z)}{dz} = \phi'(z)$$

Hence, the iteration formula is given by equation 6:

EQN (6) $$Z_{n+1} = Z_n - \frac{G(Z_n)}{\phi'(Z_n)} \qquad n=0,\ 1,\ 2,\ 3,\ 4,\ \ldots$$